Bondi accretion onto cosmological black holes: a case study. Implications.

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Motivation

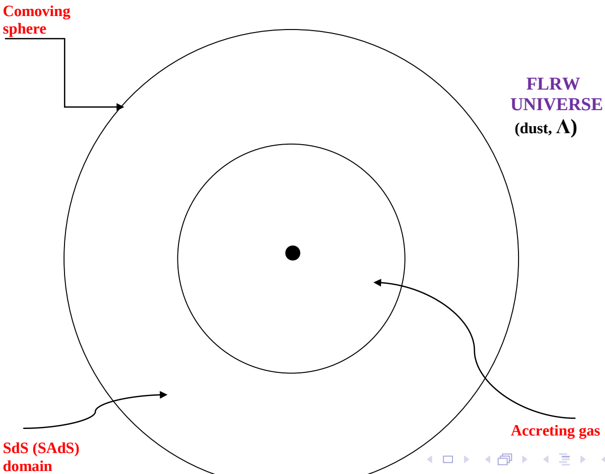
- Old question (McVittie, Jarnefelt, Einstein and Straus): is there any impact of the Hubble expansion on bound systems?
- Investigate this issue in a cosmological universe filled with dark energy, assuming steady accretion.
- Aside point: application to a modern version of the cyclic universe of Empedocles.

Accretion in an Einstein-Straus vacuole

- FLRW spacetime, with dust and dark energy,

$$ds^2 = -d\tau^2 + a^2(\tau)\left(dr^2 + r^2 d\Omega^2\right), \tag{1}$$

- Einstein and Straus (1945): vacuole within FLRW spacetime.



Accretion in an Einstein-Straus vacuole

- The Darmois-Israel gluing conditions on the first and second fundamental forms.
  Explicit solution (generalization of Schucking): *R. Balbinot, R. Bergamini and A. Comastri (1988).*
- Accretion region around a black hole: $R \leqslant R_\infty$ ($R_\infty \ll R_v$);
- outside R_∞ — the Kottler (Schwarzschild-de Sitter (SdS, $\Lambda > 0$) and Schwarzschild-anti de Sitter (SAdS, $\Lambda < 0$)) geometry

$$ds^2 = -\left(1 - \frac{2m}{R} - \frac{\Lambda}{3}R^2\right)dt^2 + \frac{dR^2}{1 - \frac{2m}{R} - \frac{\Lambda}{3}R^2} + R^2 d\Omega^2. \quad (2)$$

- In the accretion region: eqs. are approximately stationary in time intervals that are much smaller than a characteristic time $T = m/\dot{M}$.

Accretion in an Einstein-Straus vacuole

- **The metric** in the accretion zone:

$$ds^2 = -N^2dt^2 + \hat{a}dr^2 + R^2\left(d\theta^2 + \sin^2(\theta)d\phi^2\right), \qquad (3)$$

  comoving coordinates.

- The (areal radius) velocity of gas — $U = \frac{1}{N}\frac{dR}{dt}$.
  n_i — the unit normal to a coordinate(nested) sphere in the hypersurface $t = const$;
  k — the related mean curvature scalar, $k = \frac{R}{2}\nabla_i n^i = \frac{1}{\sqrt{\hat{a}}}\partial_r R$.

- **Matter**:
  $T_{\mu\nu} = (\rho + p)\,U_\mu U_\nu + pg_{\mu\nu}$, U_μ, $U_\mu U^\mu = -1$.
  The equation of state $p = (\Gamma - 1)\,\rho_0\epsilon$, ($\epsilon$ — the specific internal energy, Γ a constant);
  (isentropic accretion) $\rightarrow p = K\rho_0^\Gamma$
  $\rightarrow \rho = \rho_0 + \rho_0\epsilon = \rho_0 + K\rho_0^\Gamma/(\Gamma - 1)$.

Accretion in E-S vacuole: equations

- **Convenient description**: hydrodynamic quantities in terms of $a = \sqrt{\partial_\rho p}$, the speed of sound.

$$
\begin{aligned}
p \quad &= \rho_0 \frac{\Gamma - 1}{\Gamma} \frac{a^2}{\Gamma - 1 - a^2}, \\
\rho &= \rho_0 \frac{\Gamma - 1}{\Gamma - 1 - a^2} - p, \\
\rho_0 \quad &= \quad \rho_{0\infty} \left(\frac{a}{a_\infty} \right)^{\frac{2}{\gamma - 1}} \left(\frac{1 - \frac{a_\infty^2}{\Gamma - 1}}{1 - \frac{a^2}{\Gamma - 1}} \right)^{\frac{1}{\Gamma - 1}} . \qquad (4)
\end{aligned}
$$

Accretion in E-S vacuole: equations

- **Metric functions:**
-

$$k=\sqrt{1-\frac{2m(R)}{R}-\frac{\Lambda}{3}R^2+U^2}, \tag{5}$$

$$m(R)=m-4\pi\int_R^{R_\infty}drr^2\rho. \tag{6}$$

-

$$\partial_R U^2=-\frac{4U^2}{R}-2U^2\partial_R\ln\left(\rho_0\right) \tag{7}$$

Accretion in E-S vacuole: equations

- $$N = \tilde{C}\left(\Gamma - 1 - a^2\right). \tag{8}$$

  (from the relativistic Euler equation)
- The line element, in (t, R) coordinates

$$ds^2 = -\left(N^2 - U^2\right) dt^2 - 2\frac{NU}{k} dt dR + \frac{dR^2}{k^2} + R^2 d\Omega^2. \tag{9}$$

Accretion in E-S vacuole: equations

- # Hydrodynamic functions
- The mass accretion rate

$$\dot{M} \equiv \partial_{t_S} m(R) = 4\pi N U R^2 (\rho + p) = -4\pi U R^2 \rho_0 \tag{10}$$

satisfies $\partial_R \dot{M} = 0$.

- The system of equations (5) — (8) closes with $G_{rr} = 8\pi T_{rr}$:

$$\frac{d}{dR} \ln\left(a^2\right) = -\frac{\Gamma - 1 - a^2}{a^2 - \frac{U^2}{k^2}} \times \frac{1}{k^2 R}\left(\frac{m(R)}{R} - 2U^2 + 4\pi R^2 p - \frac{\Lambda R^2}{3}\right). \tag{11}$$

Accretion in E-S vacuole: equations

- # Transsonicity and boundary conditions
- **Transsonic accretion flows:**
- At a (sonic)sphere with the radius R_*:
  $a^2 - \frac{U^2}{k^2} = 0$
  $\frac{m(R)}{R} - 2U^2 + 4\pi R^2 p - \frac{\Lambda R^2}{3} = 0.$
  R_* — a point with two branching solutions (accretion or wind). In the accretion branch, below the sonic point the infall velocity $|U|/k$ is bigger than a, while outside the sonic sphere the converse is true.
- Boundary conditions at R_∞:

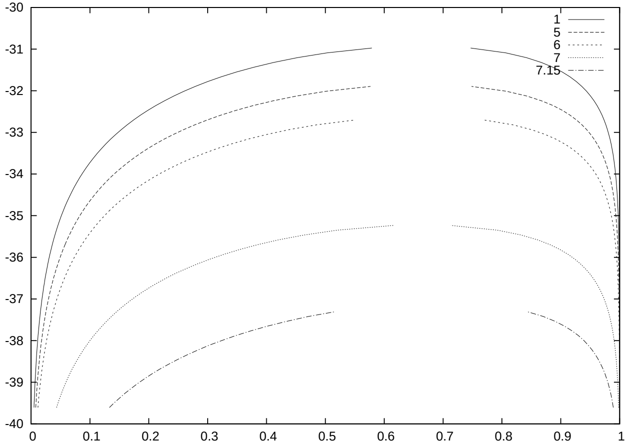
$$a_\infty^2 \gg \frac{m}{R_\infty} \gg U_\infty^2.$$

Transsonic flows: qualitative results, SDS

- $\Lambda/4\pi$ is smaller than the averaged matter density $3m/4\pi R_\infty^3$. Λ impacts the accretion mass rate $\dot{M}$, but qualitative features of the flow are not influenced.
- Large Λ implies a large Hubble expansion velocity which obstructs or (even) prohibits accretion, $\dot{M} \to 0$ with increasing Λ.
- Solutions are absent if $\Lambda R_\infty^2 \leqslant -6\frac{\Gamma-1-a_\infty^2}{\Gamma-1-a_*^2}$.

Transsonic flows: numerical solutions

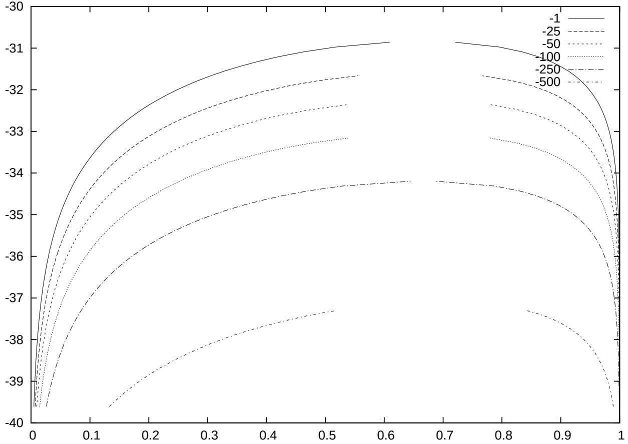
- Slns (c. 100 slns with different $\dot{M}$'s) found for $\Lambda < 7.2 \times 10^{-15}$.



Rysunek : The ordinate shows $\dot{M}$ and the abscissa shows $1 - x$, where x is the relative mass of gas in the system.. The various lines correspond to $\Lambda R_\infty^2 / 10^{-3} = 1, 5, 6, 7, 7.15$ in the increasing order from the bottom.

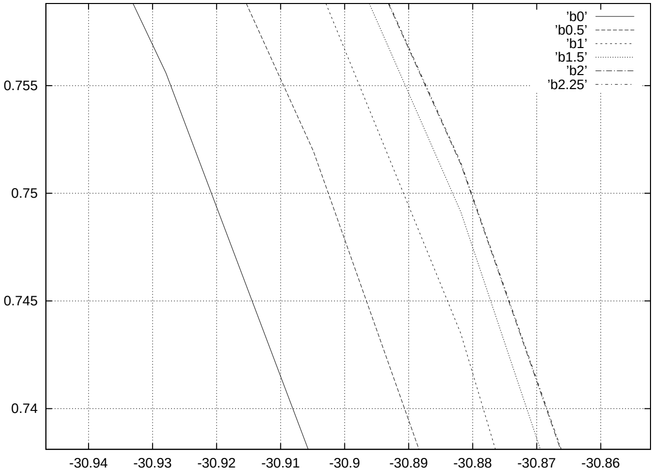
Transsonic flows: numerical solutions

- Slns (c. 100 slns with different $\dot{M}$'s) found for $\Lambda > -5 \times 10^{-13}$.



Rysunek : The ordinate shows $\dot{M}$ and the abscissa shows $1 - x$, where x is the relative mass of gas in the system. The various lines correspond to $\Lambda R_\infty^2/10^{-3} = -1, -25, -50, -100, -250, -500$ in the decreasing order from the top.

Numerical solutions: flows in SAdS.



Rysunek : The ordinate shows $1 - x$, where x is the relative mass of gas in the system, and the abscissa shows $\ln \dot{M}$. The various lines correspond to $\Lambda R_\infty^2/10^{-3} = 0, = -0.5, -1, -1.5, -2, -2.25$ in the increasing order from the bottom.

Summary and cosmological implications

- The mass accretion rate $\dot{M}$ is maximized at a $\Lambda_{max} < 0$ and decreases with the increase of $|\Lambda - \Lambda_{max}|$.
- $\Omega_\Lambda = 10^3\Omega_m \to 7 \times 10^3\Omega_m$
  $\to \dot{M}$ decreases by seven orders in magnitude
- The steady accretion onto black holes is halted during the inflation era and after 10^{12} years.

Conclusions

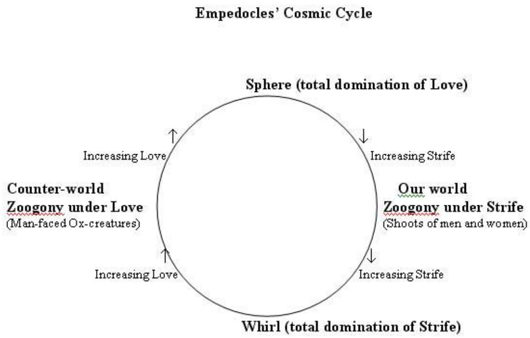
- The absence of steady spherical accretion in principle does not mean that the formation of structures (including, say, the merger of two compact objects or formation of a black hole) is prohibited.
  But:
- **Conjecture:**
  dark energy damps any accreting/(structure building) processes.
- *Answer to McVittie, Jarnefelt, Einstein-Straus*:
  the global (Λ-induced Hubble expansion) acts onto the local (accretion).
- Λ-measurements via local cosmic accretion laboratories?

Empedocles: Cyclic Cosmology

Empedocles, c. 490–430 BC, a Greek pre-Socratic philosopher and a citizen of Acragas, a Greek city in Sicily.

Cosmology of Empedocles

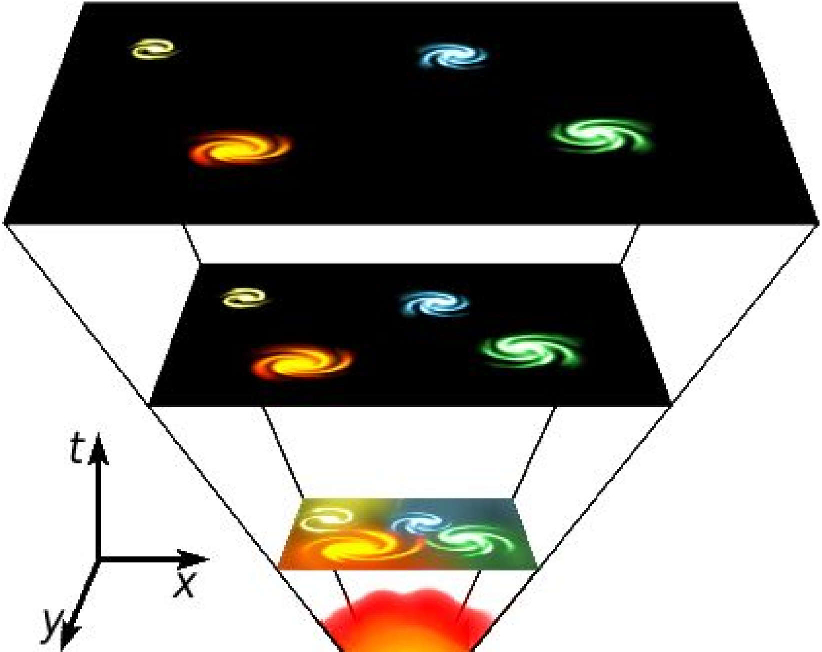
- Helge Kragh, *Ancient Greek-Roman Cosmology: Infinite, Eternal, Finite, Cyclic, and Multiple Universes, Journal of Cosmology, 2010, Vol 9, 2172-2178.* :



Cosmology of Empedocles in modern language

- LOVE = "BIG BANG" (or RICCI SCALAR CURVATURE DOMINATION)
- STRIFE = WEYL CURVATURE DOMINATION

Cosmology of Empedocles in modern language



CCC: Conformal Cyclic Cosmology

- The aeon scenario:
- Hubble flow (Ricci tensor domination);
- Conventional formation of structures;
- in late evolution: merger of stars, stones et. and formation of black holes;
- partial merger of black holes, Hawking evaporation of all black holes (with a loss of information);
- Conjecture: each aeon ends at a spatial hypersurface of vanishing Weyl curvature, that in turn begins a new aeon.

Conclusions

- Λ damps the mass accretion rate and ...
- completely stops the steady accretion onto black holes during the inflation era and after 10^{12} years.
- This *(strongly?)* suggests that only a part of the material content of the Universe would find a way into black holes.
- Therefore the necessary condition for the formation of a "next generation aeon" probably cannot be satisfied.