More on CCC: Spała 2014

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Abstract

Based on arXiv:1309.7248, ‘The equations of CCC’, and unpublished work with Laszlo Szabados.

I suggest a slight modification to Penrose’s prescription for CCC and show how this works out for FRW cosmologies, and I consider the problem of defining mass at a space-like $\mathcal{I}^+$.

The basic picture

Motivating CCC:

- as a singularity of a Lorentz manifold. the Big Bang B was *very special*;
- the specialness was something like *finite or zero Weyl tensor* at B;
- with a positive Λ there will automatically be a space-like future boundary $\mathcal{I}$ at which the Weyl tensor automatically vanishes.

Whence the outrageous suggestion (RP 2005):

Identify $\mathcal{I}$ of one *aeon* with B of a following one: the *conformal* metric is *cyclic*.

The basic picture continued

Concentrate on two particular aeons, so that's three manifolds-with-metric:

- the previous aeon $(\hat{M}, \hat{g})$;
- the present aeon $(\check{M}, \check{g})$;
- the conformal extension (M, g) of both, so that

$$\hat{g} = \hat{\Omega}^2 g, \quad \check{g} = \check{\Omega}^2 g;$$

and then $M = \hat{M} \cup \check{M} \cup \Sigma$ where Σ is the common boundary:

$$\Sigma = \{\hat{\Omega}^{-1} = 0\} = \{\check{\Omega} = 0\}.$$

The basic picture continued

Then following Penrose, the assumptions of CCC entail:

- Σ is $\mathcal{I}^+$ for $\hat{M}$ and the big bang for $\check{M}$;
- RP's *reciprocal hypothesis*: $\check{\Omega}\hat{\Omega} = -1$, which is essentially a gauge condition, and now $\check{g} = \hat{\Omega}^{-4}\hat{g}$;
- the matter content in $\hat{M}$ near Σ is radiation fluid plus positive cosmological constant $\hat{\Lambda}$ (possibly plus dynamically unimportant Maxwell fields which nevertheless go right through $\mathcal{I}^+$).

The basic picture continued

Then necessarily

- since $\check{g} = \hat{\Omega}^{-4}\hat{g}$, the matter content in $\check{M}$ near Σ will be determined from the previous aeon by $\hat{g}$ and $\hat{\Omega}$; it should be roughly the same as in $\hat{M}$ but with an extra field to represent dark matter;
- the Weyl tensors satisfy

$$\hat{C}_{abc}{}^{d} = \check{C}_{abc}{}^{d} = C_{abc}{}^{d} \text{ and vanish at } \Sigma.$$

Now the key question is:

- How does one specify a unique $\hat{\Omega}$ given $\hat{g}$?

as without a prescription for a unique $\hat{\Omega}$ one cannot get $\check{M}$ from $\hat{M}$.

The strategy for making CCC models

- Choose the matter model you want in $\hat{M}$;
- seek a unique prescription for $\hat{\Omega}$;
- use the conformal rescaling to define the Einstein tensor $\check{G}_{ab}$ in $\check{M}$;
- and seek to interpret it.

The Starobinsky expansion

Starobinsky (1983) considered metrics:

$$g = dt^2 - h_{ij}(t, x^i)dx^i dx^j$$

$$= dt^2 - e^{2Ht}(a_{ij} + e^{-2Ht}b_{ij} + e^{-3Ht}c_{ij} + \ldots)dx^i dx^j,$$

where $\Lambda = 3H^2$.

- these have 'full asymptotics' (Rendall 2003);
- $(a_{ij},\ c_{ij})$ are free data (there may be more);
- Friedrich's solutions (with data at $\mathcal{I}^+$, 1986, 1991) have this expansion.

Ask: what is the gauge freedom in the Starobinsky expansion?

The Starobinsky expansion continued

Gauge freedom: shift $t \to \tilde{t} = t - \phi(x^i)$ together with a redefinition of the comoving space coordinates; this gives

$$a_{ij} \to \tilde{a}_{ij} = e^{2H\phi} a_{ij}, \quad c_{ij} \to \tilde{c}_{ij} = e^{-H\phi} c_{ij}.$$

Our strategy will be to take $\hat{g}_{ab}$ in the Starobinski form and seek a unique prescription for $\hat{\Omega}$ by constraining a_{ij}.

The example of FRW

- the metric is $g = dt^2 - R(t)^2 d\sigma_k^2$;
- with a radiation fluid source we have $\rho R^4 = m$, a constant;
- the Friedmann equation is

$$\left(\frac{dR}{d\tau}\right)^2 = \frac{m}{3} - kR^2 + \frac{\Lambda}{3}R^4,$$

  where $d\tau = dt/R$ (defining *conformal time*).

Take this for $\hat{g}$ and put hats on everything: $\hat{R}$, $\hat{t}$, $\hat{\rho}$, $\hat{\Lambda}$, ...

Now try the obvious: $\hat{\Omega} = c_1 \hat{R}$ for some constant c_1 to be fixed.

The example of FRW continued

Then

$$\check{g} = \hat{\Omega}^{-4}\hat{g} = d\check{t}^2 - \check{R}(\check{t})^2 d\sigma_k^2,$$

with

$$\check{R} = c_1^2 \hat{R}^{-1}, \quad d\tau = d\hat{t}/\hat{R} = d\check{t}/\check{R}.$$

So

- $\check{g}$ is again in the FRW form;
- with the choice $c_1 = (\hat{\Lambda}/\hat{m})^{1/4}$ the Friedmann equation transforms from a hatted to a checked version with

$$\check{m} = \hat{m}, \quad \check{\Lambda} = \hat{\Lambda};$$

- and the two aeons are diffeomorphic i.e the same solutions of the EFEs.

How close can we stay to this case when there is some Weyl curvature?

Finding a unique $\hat{\Omega}$

With $\hat{g}$ in Starobinski form, expand $\hat{\Omega}^{-1}$ as

$$\phi := \hat{\Omega}^{-1} = e^{-H\hat{t}}\phi_1 + e^{-2H\hat{t}}\phi_2 + e^{-3H\hat{t}}\phi_3 + \dots .$$

The scalar curvature s of the intermediate metric g satisfies the equation

$$\hat{\Box}\phi + 2H^2\phi = \frac{1}{6}s\phi^3.$$

If $s = 12H^2$ this is Penrose's *phantom field equation*, but we can leave s as a constant to be chosen later and solve this equation term by term.

Finding a unique $\hat{\Omega}$ continued

With

$$\phi := \hat{\Omega}^{-1} = e^{-H\hat{t}}\phi_1 + e^{-2H\hat{t}}\phi_2 + e^{-3H\hat{t}}\phi_3 + \dots$$

claim

- ϕ_1 and ϕ_2 are freely specifiable;
- subsequent ϕ_n are determined;

while the example of FRW suggests

- choose $\phi_2 = 0$ (which is Penrose's *Delayed Rest Mass Hypothesis*) and choose ϕ_1 to make the metric of $\mathcal{I}^+$ (which is $\phi_1^2 a_{ij}$) have constant scalar curvature;
- set $s = s^{\mathcal{I}^+}$ with $s^{\mathcal{I}^+}$ the scalar curvature of the metric of $\mathcal{I}^+$ (rather than $s = 12H^2$).

These choices fix $\hat{\Omega}$ or ϕ up to a single constant which we may hope to fix by demanding $\check{\Lambda} = \hat{\Lambda}$.

The matter content after the bang

The rescaling formula for the Ricci tensor with $\check{g} = \hat{\Omega}^{-4}\hat{g}$ is

$$\hat{R}_{ab} = \check{R}_{ab} + 2\check{\nabla}_a \Upsilon_b - 2\Upsilon_a \Upsilon_b + \check{g}_{ab}\check{g}^{ef}(\check{\nabla}_e \Upsilon_f + 2\Upsilon_e \Upsilon_f)$$

with $\Upsilon_a = 2\partial_a \log \hat{\Omega}$. The Einstein equations in $\hat{M}$ are

$$\hat{R}_{ab} = -\kappa \hat{T}_{ab} + \hat{\Lambda}\hat{g}_{ab},$$

where

$$\hat{T}_{ab} = \frac{1}{3}\hat{\rho}(4\hat{u}_a\hat{u}_b - \hat{g}_{ab}).$$

Now solve for $\check{G}_{ab}$!

The matter content after the bang continued

To preserve the conservation equation we define

$$\check{T}_{ab} = \hat{\Omega}^4\,\hat{T}_{ab} = \phi^{-4}\,\hat{T}_{ab}$$

and then

$$\check{G}_{ab} = -\kappa\phi^4\,\check{T}_{ab} + \frac{4}{\phi}\check{\nabla}_a\check{\nabla}_b\phi + \frac{4}{\phi^2}\phi_a\phi_b + \left(8\frac{|\check{\nabla}\phi|^2}{\phi^2} - 4\frac{\check{\Box}\phi}{\phi} - \frac{\hat{\Lambda}}{\phi^4}\right)\check{g}_{ab}.$$

Does this give sensible answers? Examples (expanding $\check{g}$ as a power series) indicate that it does.

What is the physical content of CCC?

In the sense of observable consequences:

- Circles in the CMB (P&G, MNR, AMN, width?);
- BICEP2 and magnetic fields (which go through $\mathcal{I}^+$);
- inflation happening before the Bang?

These motivate the question of how to think about energy coming through the Bang, and therefore how to think about mass at $\mathcal{I}^+$...

Mass at $\mathcal{I}^+$ with $\Lambda > 0$

Can this be defined, by analogy with other asymptotic measures of mass, energy, momentum, ...?

Think about linear theory in de Sitter space, then:

- there are plenty of Killing vectors in de Sitter, giving rise to conserved quantities, but
- there are no Killing vectors in de Sitter which are time-like across a complete space-like hypersurface;
- in fact all Killing vectors are tangent to $\mathcal{I}^+$;
- but there are time-like conformal Killing vectors, which produce conserved currents from trace-free T_{ab}.

And this continues to hold in FRW with $\Lambda > 0$....

Mass at $\mathcal{I}^+$ continued

In FRW with $\Lambda > 0$:

- with $g = dt^2 - R(t)^2 d\sigma_k^2$, **claim** $K^a = Ru^a$ is a CKV;
- corresponding current is $J_a := T_{ab}K^b = \rho R u_a$ and is conserved for radiation equation of state,
- when also $\rho_0 := \rho R^4 =$ const. so that
- $$M(\mathbf{V}) := \int_\Sigma J = \rho_0 V_\sigma(\mathbf{V})$$

  where Σ is a space-like surface spanning a volume $\mathbf{V}$ on $\mathcal{I}^+$ and V_σ is volume measured by $d\sigma_k^2$, the metric of $\mathcal{I}^+$.

which in turn continues to work with the Starobinski metric:

Mass at $\mathcal{I}^+$ continued

With the Starobinski metric

- set $J_a := \rho R u_a$ with the modified definition $R \sim (\det h_{ij} / \det a_{ij})^{1/6}$;
- then this is conserved for radiation e.o.s. and again $\rho_0 := \rho R^4 =$ const., so that
- $M(\mathbf{V}) := \int_\Sigma J = \rho_0 V_a(\mathbf{V})$, when V_a is volume measured by a_{ij}, the metric of $\mathcal{I}^+$.

This definition evidently omits contributions from gravitational radiation to total energy. For that, try something else:

Mass at $\mathcal{I}^+$ continued: Penrose's quasi-local quantities

Recall this construction:

- given a space-like, topologically-spherical two surface S find the *two-surface twistors* of S – these are a $\mathbb{C}^4$ of special spinor fields $\omega_i^A(x) \sim Z_i^\alpha \in \mathbb{T}(S)$ determined on S by the first and second fundamental forms of S;
- construct the bilinear $A_{\alpha\beta} Z_1^\alpha Z_2^\beta := \int_S R_{abCD} \omega_1^C \omega_2^D dS^{ab}$ from the Riemann tensor at S;
- then $A_{\alpha\beta}$ defines the quasi-local kinematic quantities at S.

This construction reproduces the standard definitions of asymptotic momenta in asymptotically-flat and asymptotically-anti de Sitter space-times, and links up with the Witten argument to prove positivity.

Work in progress...

Nothing new...

"And I saw a new heaven and a new earth: for the first heaven and the first earth were passed away; and there was no more sea."

The Revelation of St. John the Divine; Chapter 21, Verse 1.