Towards solving generic cosmological singularity problem

Włodzimierz Piechocki

Department of Fundamental Research
National Centre for Nuclear Research
Warsaw, Poland

Quantum Cosmology Group:
Michał Artymowski, Ewa Czuchry, Orest Hrycyna, Marek Kuś, Przemek Małkiewicz, Kuba Mielczarek, WP, Adam Sawicki

http://quantcosmo.ncbj.pl/

OUTLINE

Introduction

Evidence for the existence of the cosmological singularity

- observational cosmology:
  the Universe has been expanding for almost 14 billion years (emerged from a state with extremely high energy densities of physical fields)
- theoretical cosmology:
  almost all known general relativity models of the Universe: (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL) predict the existence of cosmological singularities

Existence of the cosmological singularities in solutions to GR means that this classical theory is incomplete

Expectation: quantization may heal the cosmological singularity

Introduction

Evidence for the existence of the cosmological singularity

- observational cosmology:
  the Universe has been expanding for almost 14 billion years (emerged from a state with extremely high energy densities of physical fields)
- theoretical cosmology:
  almost all known general relativity models of the Universe: (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL) predict the existence of cosmological singularities

Existence of the cosmological singularities in solutions to GR means that this classical theory is incomplete

Expectation: quantization may heal the cosmological singularity

Introduction

Evidence for the existence of the cosmological singularity

- observational cosmology:
  the Universe has been expanding for almost 14 billion years (emerged from a state with extremely high energy densities of physical fields)
- theoretical cosmology:
  almost all known general relativity models of the Universe: (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL) predict the existence of cosmological singularities

Existence of the cosmological singularities in solutions to GR means that this classical theory is incomplete

Expectation: quantization may heal the cosmological singularity

Introduction

Evidence for the existence of the cosmological singularity

- observational cosmology:
  the Universe has been expanding for almost 14 billion years (emerged from a state with extremely high energy densities of physical fields)
- theoretical cosmology:
  almost all known general relativity models of the Universe: (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL) predict the existence of cosmological singularities

Existence of the cosmological singularities in solutions to GR means that this classical theory is incomplete

Expectation: quantization may heal the cosmological singularity

Introduction

Evidence for the existence of the cosmological singularity

- observational cosmology:
  the Universe has been expanding for almost 14 billion years
  (emerged from a state with extremely high energy densities of physical fields)
- theoretical cosmology:
  almost all known general relativity models of the Universe:
  (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL)
  predict the existence of cosmological singularities

Existence of the cosmological singularities in solutions to GR means that this classical theory is incomplete

Expectation: quantization may heal the cosmological singularity

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase ⇄ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition:
  quantum phase ⇄ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition:
  quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition:
  quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition:
  quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition:
  quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition:
  quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase $\rightleftarrows$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
  - What is the origin of primordial gravitational waves?
  - What is the origin of EM field?
  - What is the origin of matter?
- If the notion of time is well defined:
  - How long had the quantum phase lasted?
  - What was before the quantum phase?

Introduction (cont)

- Canonical quantization based on the Holst action and loop geometry
  - Dirac's LQC[1] := 'first quantize then impose constraints'
  - RPS LQC[2] := 'first solve constraints then quantize'
- Coherent states[3] and canonical[4] quantizations based on the Hilbert-Einstein action

[1] Ashtekar, Bojowald, Lewandowski, Rovelli, Thiemann, . . .
[2] P. Dzierżak, P. Małkiewicz, J. Mielczarek, W.P., . . .
[3] H. Bergeron, J.P. Gazeau, P. Małkiewicz, W.P.,. . .
[4] D.M. Chitre, C.W. Misner, M.P. Ryann, . . .

Introduction (cont)

- Canonical quantization based on the Holst action and loop geometry
  - Dirac's LQC[1] := 'first quantize then impose constraints'
  - RPS LQC[2] := 'first solve constraints then quantize'
- Coherent states[3] and canonical[4] quantizations based on the Hilbert-Einstein action

[1] Ashtekar, Bojowald, Lewandowski, Rovelli, Thiemann, ...
[2] P. Dzierżak, P. Małkiewicz, J. Mielczarek, W.P., ...
[3] H. Bergeron, J.P. Gazeau, P. Małkiewicz, W.P.,...
[4] D.M. Chitre, C.W. Misner, M.P. Ryann, ...

Introduction (cont)

- Canonical quantization based on the Holst action and loop geometry
  - Dirac's LQC[1] := 'first quantize then impose constraints'
  - RPS LQC[2] := 'first solve constraints then quantize'
- Coherent states[3] and canonical[4] quantizations based on the Hilbert-Einstein action

---

[1] Ashtekar, Bojowald, Lewandowski, Rovelli, Thiemann, . . .
[2] P. Dzierżak, P. Małkiewicz, J. Mielczarek, W.P., . . .
[3] H. Bergeron, J.P. Gazeau, P. Małkiewicz, W.P.,. . .
[4] D.M. Chitre, C.W. Misner, M.P. Ryann, . . .

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

---

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons[5] up to the energy 5×10^{17} GeV
- Existence of primordial gravity waves: B type tensor modes[6]
- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[5] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

[6] BICEP 2 detection; J. Mielczarek, "Inflationary power spectra with quantum holonomy corrections", JCAP **1403** (2014) 048

Main papers:

[1] P. Dzierżak, P. Małkiewicz and W. P.,
'Turning big bang into big bounce: I. Classical dynamics',
Phys. Rev. D **80**, 104001 (2009).
[2] P. Małkiewicz and W. P.,
'Turning big bang into big bounce: II. Quantum dynamics'.
Class. Quant. Grav. **27** (2010) 225018.
[3] P. Małkiewicz and W. P.,
'Energy scale of the Big Bounce',
Phys. Rev. D **80**, 063506 (2009).
[4] J. Mielczarek and W. P.,
'Observables for FRW model with cosmological constant'.
Phys. Rev. D **82** (2010) 043529.
[5] J. Mielczarek and W. P.,
'Evolution in bouncing quantum cosmology',
Class. Quant. Grav. **29** (2012) 065022.
[6] J. Mielczarek and W. P.,
'Gaussian state for the bouncing quantum cosmology ',
Phys. Rev. D **86** (2012) 083508.
[7] J-P. Gazeau, J. Mielczarek and W. P.,
'Quantum states of the bouncing universe',
Phys. Rev. D **87**, 123508 (2013).

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Challenge in Cosmology:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario is thought to be generic solution to GR near CS
  - does not rely on any symmetry conditions;
  - corresponds to non-zero measure subset of all initial conditions;
  - solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Quantization of the Bianchi IX model:

- Dynamics of Bianchi-IX is the best prototype for the BKL scenario[7]
- Questions to answer:
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
  - What is the quantum generation of primordial GW for the Bianchi IX model?
- Successful quantization of the Bianchi IX model would open the door to the quantization of the BKL scenario.

[7] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, "Oscillatory approach to a singular point in the relativistic cosmology", Adv. Phys. **19** (1970) 525.

Quantization of the Bianchi IX model:

- Dynamics of Bianchi-IX is the best prototype for the BKL scenario[7]
- Questions to answer:
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
  - What is the quantum generation of primordial GW for the Bianchi IX model?
- Successful quantization of the Bianchi IX model would open the door to the quantization of the BKL scenario.

[7] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, "Oscillatory approach to a singular point in the relativistic cosmology", Adv. Phys. **19** (1970) 525.

Quantization of the Bianchi IX model:

- Dynamics of Bianchi-IX is the best prototype for the BKL scenario[7]
- Questions to answer:
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
  - What is the quantum generation of primordial GW for the Bianchi IX model?
- Successful quantization of the Bianchi IX model would open the door to the quantization of the BKL scenario.

[7] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, "Oscillatory approach to a singular point in the relativistic cosmology", Adv. Phys. **19** (1970) 525.

Quantization of the Bianchi IX model:

- Dynamics of Bianchi-IX is the best prototype for the BKL scenario[7]
- Questions to answer:
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
  - What is the quantum generation of primordial GW for the Bianchi IX model?
- Successful quantization of the Bianchi IX model would open the door to the quantization of the BKL scenario.

[7] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, "Oscillatory approach to a singular point in the relativistic cosmology", Adv. Phys. **19** (1970) 525.

Quantization of the Bianchi IX model:

- Dynamics of Bianchi-IX is the best prototype for the BKL scenario[7]
- Questions to answer:
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
  - What is the quantum generation of primordial GW for the Bianchi IX model?
- Successful quantization of the Bianchi IX model would open the door to the quantization of the BKL scenario.

[7] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, "Oscillatory approach to a singular point in the relativistic cosmology", Adv. Phys. **19** (1970) 525.

Quantization of the Bianchi IX model:

- Dynamics of Bianchi-IX is the best prototype for the BKL scenario[7]
- Questions to answer:
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
  - What is the quantum generation of primordial GW for the Bianchi IX model?
- Successful quantization of the Bianchi IX model would open the door to the quantization of the BKL scenario.

[7] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, “Oscillatory approach to a singular point in the relativistic cosmology”, Adv. Phys. **19** (1970) 525.

The world-interval of spacetime

Presented results are based on the forthcoming paper[8]

- The line element of the homogenous Bianchi type model in spacetime $\mathcal{M} \mapsto \Sigma \times \mathbb{R}$, where Σ is spacelike

$$s^2 = -\mathcal{N}(t)\, t^2 + \sum_i q_i(t)\omega^i \otimes \omega^i \,, \tag{1}$$

where ω^i are 1-forms on Σ invariant with respect to the action of a simply transitive group of motions on the leaf and subject to

$$\omega^i = \frac{1}{2} C^i_{jk} \omega^j \wedge \omega^k \,, \tag{2}$$

where C^i_{jk} are structure constants of the corresponding Lie algebra.

[8] H. Bergeron, E. Czuchry, J-P. Gazeau, P. Małkiewicz, and W.P.

Classical Hamiltonian

In the Misner type parametrization of phase space we have

$$\mathcal{H} = \mathcal{N}(t) \left(\frac{2\pi G}{3c^2 a^3} \left(a^2 p_a^2 - p_+^2 - p_-^2 \right) - \frac{c^4}{32\pi G} a W_n(\beta_\pm) \right) \approx 0, \qquad (3)$$

where $(a, \beta\pm; p_a, p_\pm)$ are canonical variables of the kinematical phase space.

Well known homogeneous models can be obtained as follows:

- FRW, by taking $W_n(\beta_\pm) = 0$ and $p_\pm = 0$;
- Bianchi-I, corresponds to $W_n(\beta_\pm) = 0$;
- Bianchi-II, has $W_n(\beta_\pm) = n^2 e^{8\beta_+}$ and $n > 0$.

Classical Hamiltonian

In the Misner type parametrization of phase space we have

$$\mathcal{H} = \mathcal{N}(t)\left(\frac{2\pi G}{3c^2a^3}\left(a^2p_a^2 - p_+^2 - p_-^2\right) - \frac{c^4}{32\pi G}aW_n(\beta_\pm)\right) \approx 0, \qquad (3)$$

where $(a, \beta\pm; p_a, p_\pm)$ are canonical variables of the kinematical phase space.

Well known homogeneous models can be obtained as follows:

- FRW, by taking $W_n(\beta_\pm) = 0$ and $p_\pm = 0$;
- Bianchi-I, corresponds to $W_n(\beta_\pm) = 0$;
- Bianchi-II, has $W_n(\beta_\pm) = n^2 e^{8\beta_+}$ and $n > 0$.

Classical Hamiltonian (cont)

The Bianchi IX model is defined by

$$W_n(\beta_\pm) = n^2 e^{-4\beta_+}\left(\left(e^{6\beta_+} - 2\cosh(2\sqrt{3}\beta_-)\right)^2 - 4\right), \; n > 0. \qquad (4)$$

The potential W_n is bounded from below and reaches its (absolute) minimal value at $\beta_\pm = 0$, with $W_n(0) = -3n^2$.
W_n has $\mathbb{C}_{3v}$ symmetry and is asymptotically confined except for three directions:

(i) $\beta_- = 0$, $\beta_+ \to -\infty$,
(ii) $\beta_+ = \beta_-/\sqrt{3}$, $\beta_- \to +\infty$,
(iii) $\beta_+ = -\beta_-/\sqrt{3}$, $\beta_- \to -\infty$,

where $W_n \to 0$.

Classical Hamiltonian (cont)

The Bianchi IX model is defined by

$$W_n(\beta_\pm) = n^2 e^{-4\beta_+}\left(\left(e^{6\beta_+} - 2\cosh(2\sqrt{3}\beta_-)\right)^2 - 4\right), \;\; n > 0. \qquad (4)$$

The potential W_n is bounded from below and reaches its (absolute) minimal value at $\beta_\pm = 0$, with $W_n(0) = -3n^2$.

W_n has $\mathbb{C}_{3v}$ symmetry and is asymptotically confined except for three directions:

(i) $\beta_- = 0$, $\beta_+ \to -\infty$,

(ii) $\beta_+ = \beta_-/\sqrt{3}$, $\beta_- \to +\infty$,

(iii) $\beta_+ = -\beta_-/\sqrt{3}$, $\beta_- \to -\infty$,

where $W_n \to 0$.

Classical Hamiltonian (cont)

The Bianchi IX model is defined by

$$W_n(\beta_\pm) = n^2 e^{-4\beta_+}\left(\left(e^{6\beta_+} - 2\cosh(2\sqrt{3}\beta_-)\right)^2 - 4\right),\ \ n > 0. \tag{4}$$

The potential W_n is bounded from below and reaches its (absolute) minimal value at $\beta_\pm = 0$, with $W_n(0) = -3n^2$.
W_n has $\mathbb{C}_{3v}$ symmetry and is asymptotically confined except for three directions:

(i) $\beta_- = 0$, $\beta_+ \to -\infty$,

(ii) $\beta_+ = \beta_-/\sqrt{3}$, $\beta_- \to +\infty$,

(iii) $\beta_+ = -\beta_-/\sqrt{3}$, $\beta_- \to -\infty$,

where $W_n \to 0$.

Classical Hamiltonian (cont)

Redefining the phase space variables, to highlight possible approximation, by introducing the canonical pair $(q = a^{3/2}, p = 2p_a/(3\sqrt{a}))$ we get

$$\mathcal{H} = \mathcal{N}(t) \left(\frac{2\pi G}{3c^2} \left(\frac{9}{4} p^2 - \frac{p_+^2 + p_-^2}{q^2} \right) - \frac{c^4}{32\pi G} q^{2/3} W_n(\beta_\pm) \right). \quad (5)$$

It results from Eq. (5) that near the singularity, $q = 0$, we may treat q as heavy degree of freedom, and $\beta_\pm$ as light degrees of freedom. It is so because 'the mass' of the $\beta_\pm$ behaves as q^2, while 'the mass' of q is fixed.

Therefore, we may quantize our system by using the adiabatic approximation.

Classical Hamiltonian (cont)

Redefining the phase space variables, to highlight possible approximation, by introducing the canonical pair $(q = a^{3/2}, p = 2p_a/(3\sqrt{a}))$ we get

$$\mathcal{H} = \mathcal{N}(t)\left(\frac{2\pi G}{3c^2}\left(\frac{9}{4}p^2 - \frac{p_+^2 + p_-^2}{q^2}\right) - \frac{c^4}{32\pi G}q^{2/3}W_n(\beta_\pm)\right). \quad (5)$$

It results from Eq. (5) that near the singularity, $q = 0$, we may treat q as heavy degree of freedom, and $\beta_\pm$ as light degrees of freedom. It is so because 'the mass' of the $\beta_\pm$ behaves as q^2, while 'the mass' of q is fixed.

Therefore, we may quantize our system by using the adiabatic approximation.

Classical Hamiltonian (cont)

Redefining the phase space variables, to highlight possible approximation, by introducing the canonical pair $(q = a^{3/2}, p = 2p_a/(3\sqrt{a}))$ we get

$$\mathcal{H} = \mathcal{N}(t)\left(\frac{2\pi G}{3c^2}\left(\frac{9}{4}p^2 - \frac{p_+^2 + p_-^2}{q^2}\right) - \frac{c^4}{32\pi G}q^{2/3}W_n(\beta_\pm)\right). \tag{5}$$

It results from Eq. (5) that near the singularity, $q = 0$, we may treat q as heavy degree of freedom, and $\beta_\pm$ as light degrees of freedom. It is so because 'the mass' of the $\beta_\pm$ behaves as q^2, while 'the mass' of q is fixed.

Therefore, we may quantize our system by using the adiabatic approximation.

Classical Hamiltonian (cont)

For the purpose of the adiabatic quantization:

$$\mathcal{H} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}p^2 - \mathcal{H}_\pm\right), \tag{6}$$

where

$$\mathcal{H}_\pm := \frac{2\pi G}{3c^2q^2}(p_+^2 + p_-^2) + \frac{c^4}{32\pi G}q^{2/3}W_n(\beta_\pm)\,. \tag{7}$$

- $\beta_\pm = 0 = p_\pm$ corresponds to the classical ground state of the anisotropy Hamiltonian $\mathcal{H}_\pm$. Thus, FRW may be treated as a special case of Bianchi-IX , where the anisotropy degrees of freedom are frozen in their (classical) ground state.
- We cannot quantize the FRW model alone, because we should take into account the effect of quantum 'zero point energy' generated by the quantized anisotropy degrees of freedom of the Bianchi-IX model.

Classical Hamiltonian (cont)

For the purpose of the adiabatic quantization:

$$\mathcal{H} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}p^2 - \mathcal{H}_\pm\right), \tag{6}$$

where

$$\mathcal{H}_\pm := \frac{2\pi G}{3c^2 q^2}(p_+^2 + p_-^2) + \frac{c^4}{32\pi G}q^{2/3}W_n(\beta_\pm)\,. \tag{7}$$

- $\beta_\pm = 0 = p_\pm$ corresponds to the classical ground state of the anisotropy Hamiltonian $\mathcal{H}_\pm$. Thus, FRW may be treated as a special case of Bianchi-IX , where the anisotropy degrees of freedom are frozen in their (classical) ground state.
- We cannot quantize the FRW model alone, because we should take into account the effect of quantum 'zero point energy' generated by the quantized anisotropy degrees of freedom of the Bianchi-IX model.

Classical Hamiltonian (cont)

For the purpose of the adiabatic quantization:

$$\mathcal{H} = \mathcal{N}(t) \left(\frac{3\pi G}{2c^2} p^2 - \mathcal{H}_\pm \right), \tag{6}$$

where

$$\mathcal{H}_\pm := \frac{2\pi G}{3c^2 q^2}(p_+^2 + p_-^2) + \frac{c^4}{32\pi G} q^{2/3} W_n(\beta_\pm) \, . \tag{7}$$

- $\beta_\pm = 0 = p_\pm$ corresponds to the classical ground state of the anisotropy Hamiltonian $\mathcal{H}_\pm$. Thus, FRW may be treated as a special case of Bianchi-IX , where the anisotropy degrees of freedom are frozen in their (classical) ground state.
- We cannot quantize the FRW model alone, because we should take into account the effect of quantum ‘zero point energy’ generated by the quantized anisotropy degrees of freedom of the Bianchi-IX model.

Quantum Hamiltonian

In what follows we apply the modified Dirac quantization method:

- quantizing $\mathcal{H}$ (all degrees of freedom) and get $\hat{\mathcal{H}}$,
- finding semi-classical expression $\check{\mathcal{H}}$ of $\hat{\mathcal{H}}$,
- implementing constraint $\check{\mathcal{H}} = 0$,
- making adiabatic approximation.

Since $(q, p) \in \mathbb{R}_+ \times \mathbb{R}$ and $(\beta_\pm, p_\pm) \in \mathbb{R}^4$:

- we apply affine coherent states quantization to (q, p),
- we apply canonical quantization to $(\beta_\pm, p_\pm)$

Quantum Hamiltonian

In what follows we apply the modified Dirac quantization method:

- quantizing $\mathcal{H}$ (all degrees of freedom) and get $\hat{\mathcal{H}}$,
- finding semi-classical expression $\check{\mathcal{H}}$ of $\hat{\mathcal{H}}$,
- implementing constraint $\check{\mathcal{H}} = 0$,
- making adiabatic approximation.

Since $(q, p) \in \mathbb{R}_+ \times \mathbb{R}$ and $(\beta_\pm, p_\pm) \in \mathbb{R}^4$:

- we apply affine coherent states quantization to (q, p),
- we apply canonical quantization to $(\beta_\pm, p_\pm)$

Quantum Hamiltonian (cont)

The quantum Hamiltonian $\hat{\mathcal{H}}$ reads

$$\hat{\mathcal{H}} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(\hat{p}^2 + \frac{\hbar^2\mathfrak{K}_1}{\hat{q}^2}\right) - \frac{2\pi G}{3c^2}\mathfrak{K}_2\frac{\hat{p}_+^2 + \hat{p}_-^2}{\hat{q}^2} - \frac{c^4}{32\pi G}\mathfrak{K}_3\,\hat{q}^{2/3}W_n(\hat{\beta}_\pm \tag{8}$$

where the $\mathfrak{K}_i$:

$$\mathfrak{K}_1 = \frac{1}{4}\left(1 + \frac{K_0(\nu)}{K_1(\nu)}\right), \quad \mathfrak{K}_2 = \left(\frac{K_2(\nu)}{K_1(\nu)}\right)^2 \quad \mathfrak{K}_3 = \frac{K_{5/3}(\nu)}{K_1(\nu)^{1/3}K_2(\nu)^{2/3}}, \tag{9}$$

and where the $K_\alpha(\nu)$ are modified Bessel functions.
$\hat{p}_\pm = -i\hbar\partial_{\beta_\pm}$, and $\hat{\beta}_\pm$ defined as $\beta_\pm$, acting on $L^2(\mathbb{R}^2, d\beta_+ d\beta_-)$;
$\hat{p} = -i\hbar\partial_q$, and $\hat{q}$ defined as q, acting on $L^2(\mathbb{R}_+, dq)$.

Semi-classical approximation

Near the singularity we have

$$\hat{\mathcal{H}}_\pm(q) = \frac{2\pi G}{3c^2}\mathfrak{K}_2\frac{\hat{p}_+^2 + \hat{p}_-^2}{q^2} + \frac{c^4}{32\pi G}\mathfrak{K}_3\, q^{2/3} W_n(\hat{\beta}_\pm)\,. \tag{10}$$

Due to the harmonic behavior of W_n near its minimum:

$$W_n(\beta_\pm) \simeq -3n^2 + 24n^2(\beta_+^2 + \beta_-^2) + o(\beta_\pm^2)\,, \tag{11}$$

the harmonic approximation to the eigen energies $E_\pm^{(N)}(q)$ is possible ($N = 0, 1, \dots$):

$$E_\pm^{(N)}(q) \simeq -\frac{3c^4}{32\pi G}\mathfrak{K}_3 q^{2/3} n^2 + \frac{\hbar c}{q^{2/3}} n\sqrt{2\mathfrak{K}_2\mathfrak{K}_3}\,(N+1)\,. \tag{12}$$

Semi-classical approximation

Near the singularity we have

$$\hat{\mathcal{H}}_\pm(q) = \frac{2\pi G}{3c^2}\mathfrak{K}_2\frac{\hat{p}_+^2 + \hat{p}_-^2}{q^2} + \frac{c^4}{32\pi G}\mathfrak{K}_3\, q^{2/3} W_n(\hat{\beta}_\pm)\,. \tag{10}$$

Due to the harmonic behavior of W_n near its minimum:

$$W_n(\beta_\pm) \simeq -3n^2 + 24n^2(\beta_+^2 + \beta_-^2) + o(\beta_\pm^2)\,, \tag{11}$$

the harmonic approximation to the eigen energies $E_\pm^{(N)}(q)$ is possible $(N = 0, 1, \dots)$:

$$E_\pm^{(N)}(q) \simeq -\frac{3c^4}{32\pi G}\mathfrak{K}_3 q^{2/3} n^2 + \frac{\hbar c}{q^{2/3}} n\sqrt{2\mathfrak{K}_2\mathfrak{K}_3}\,(N+1)\,. \tag{12}$$

Semi-classical approximation

Near the singularity we have

$$\hat{\mathcal{H}}_\pm(q) = \frac{2\pi G}{3c^2}\mathfrak{K}_2\frac{\hat{p}_+^2 + \hat{p}_-^2}{q^2} + \frac{c^4}{32\pi G}\,\mathfrak{K}_3\, q^{2/3} W_n(\hat{\beta}_\pm)\,. \tag{10}$$

Due to the harmonic behavior of W_n near its minimum:

$$W_n(\beta_\pm) \simeq -3n^2 + 24n^2(\beta_+^2 + \beta_-^2) + o(\beta_\pm^2)\,, \tag{11}$$

the harmonic approximation to the eigen energies $E_\pm^{(N)}(q)$ is possible $(N = 0, 1, \dots)$:

$$E_\pm^{(N)}(q) \simeq -\frac{3c^4}{32\pi G}\mathfrak{K}_3 q^{2/3} n^2 + \frac{\hbar c}{q^{2/3}} n\sqrt{2\mathfrak{K}_2\mathfrak{K}_3}\,(N+1)\,. \tag{12}$$

Semi-classical approximation (cont)

The Hamiltonian $\hat{\mathcal{H}}$ is now replaced by the averaged one $\hat{\mathcal{H}}_{av}$:

$$\hat{\mathcal{H}}_{av} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(\hat{p}^2 + \frac{\hbar^2 \mathfrak{K}_1}{\hat{q}^2}\right) - E_{\pm}^{(N)}(\hat{q})\right) . \qquad (13)$$

The semi-classical expressions with affine CS, is defined as

$$\check{\mathcal{H}}_{av}(q,p) = \langle \lambda q, p | \hat{\mathcal{H}}_{av} | \lambda q, p \rangle , \qquad (14)$$

where $\lambda := K_0(\nu)/K_2(\nu)$ is chosen to get $\langle \lambda q, p | \hat{q} | \lambda q, p \rangle = q$ and $\langle \lambda q, p | \hat{p} | \lambda q, p \rangle = p$.
Finally, we obtain

$$\check{\mathcal{H}}_{av} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(p^2 + \frac{\hbar^2 \mathfrak{K}_4}{q^2}\right) + \frac{3c^4}{32\pi G}\mathfrak{K}_5 q^{2/3} n^2 - \frac{\hbar c}{q^{2/3}}\mathfrak{K}_6 n(N+1)\right) , \qquad (15)$$

where $\mathfrak{K}_4, \mathfrak{K}_5, \mathfrak{K}_6$ are numerical factors.

Semi-classical approximation (cont)

The Hamiltonian $\hat{\mathcal{H}}$ is now replaced by the averaged one $\hat{\mathcal{H}}_{av}$:

$$\hat{\mathcal{H}}_{av} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(\hat{p}^2 + \frac{\hbar^2 \mathfrak{K}_1}{\hat{q}^2}\right) - E_{\pm}^{(N)}(\hat{q})\right) . \tag{13}$$

The semi-classical expressions with affine CS, is defined as

$$\check{\mathcal{H}}_{av}(q,p) = \langle \lambda q, p | \hat{\mathcal{H}}_{av} | \lambda q, p \rangle , \tag{14}$$

where $\lambda := K_0(\nu)/K_2(\nu)$ is chosen to get $\langle \lambda q, p | \hat{q} | \lambda q, p \rangle = q$ and $\langle \lambda q, p | \hat{p} | \lambda q, p \rangle = p$.
Finally, we obtain

$$\check{\mathcal{H}}_{av} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(p^2 + \frac{\hbar^2 \mathfrak{K}_4}{q^2}\right) + \frac{3c^4}{32\pi G}\mathfrak{K}_5 q^{2/3} n^2 - \frac{\hbar c}{q^{2/3}}\mathfrak{K}_6 n(N+1)\right) , \tag{15}$$

where $\mathfrak{K}_4, \mathfrak{K}_5, \mathfrak{K}_6$ are numerical factors.

Semi-classical approximation (cont)

The Hamiltonian $\hat{\mathcal{H}}$ is now replaced by the averaged one $\hat{\mathcal{H}}_{av}$:

$$\hat{\mathcal{H}}_{av} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(\hat{p}^2 + \frac{\hbar^2\mathfrak{K}_1}{\hat{q}^2}\right) - E_{\pm}^{(N)}(\hat{q})\right) . \tag{13}$$

The semi-classical expressions with affine CS, is defined as

$$\check{\mathcal{H}}_{av}(q,p) = \langle \lambda q, p|\hat{\mathcal{H}}_{av}|\lambda q, p\rangle , \tag{14}$$

where $\lambda := K_0(\nu)/K_2(\nu)$ is chosen to get $\langle \lambda q, p|\hat{q}|\lambda q, p\rangle = q$ and $\langle \lambda q, p|\hat{p}|\lambda q, p\rangle = p$.
Finally, we obtain

$$\check{\mathcal{H}}_{av} = \mathcal{N}(t)\left(\frac{3\pi G}{2c^2}\left(p^2 + \frac{\hbar^2\mathfrak{K}_4}{q^2}\right) + \frac{3c^4}{32\pi G}\mathfrak{K}_5 q^{2/3} n^2 - \frac{\hbar c}{q^{2/3}}\mathfrak{K}_6 n(N+1)\right) , \tag{15}$$

where $\mathfrak{K}_4, \mathfrak{K}_5, \mathfrak{K}_6$ are numerical factors.

Imposition of Hamiltonian constraint:

The constraint $\check{\mathcal{H}}_{av}(q,p) = 0$ reads

$$\frac{\dot{a}^2}{a^2} + k\frac{c^2}{a^2} + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^6} = \frac{8\pi G}{3c^2}\rho(a)\,, \qquad (16)$$

where

$$\mathfrak{s}_P := 2\pi G\hbar\, c^{-3}, \quad k := \frac{\mathfrak{K}_5 n^2}{4}, \quad \rho(a) := \hbar c(N+1)\frac{n\mathfrak{K}_6}{a^4}\,. \qquad (17)$$

The main features of this quantum corrected model:

- anisotropy degrees of freedom produce the ‘radiation-like’ energy density $\rho(a)$; primordial GW?
- curvature-like term $kc^2 a^{-2}$, modifies the usual FRW result;
- new repulsive potential term with a^{-6} generated by the affine CS quantization.

Imposition of Hamiltonian constraint:

The constraint $\check{\mathcal{H}}_{av}(q,p) = 0$ reads

$$\frac{\dot{a}^2}{a^2} + k\frac{c^2}{a^2} + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^6} = \frac{8\pi G}{3c^2}\rho(a)\,, \qquad (16)$$

where

$$\mathfrak{s}_P := 2\pi G\hbar\, c^{-3}, \quad k := \frac{\mathfrak{K}_5 n^2}{4}, \quad \rho(a) := \hbar c(N+1)\frac{n\mathfrak{K}_6}{a^4}\,. \qquad (17)$$

The main features of this quantum corrected model:

- anisotropy degrees of freedom produce the 'radiation-like' energy density $\rho(a)$; primordial GW?
- curvature-like term $kc^2 a^{-2}$, modifies the usual FRW result;
- new repulsive potential term with a^{-6} generated by the affine CS quantization.

Imposition of Hamiltonian constraint:

The constraint $\check{\mathcal{H}}_{av}(q,p) = 0$ reads

$$\frac{\dot{a}^2}{a^2} + k\frac{c^2}{a^2} + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^6} = \frac{8\pi G}{3c^2}\rho(a)\,, \tag{16}$$

where

$$\mathfrak{s}_P := 2\pi G\hbar\, c^{-3}, \quad k := \frac{\mathfrak{K}_5 n^2}{4}, \quad \rho(a) := \hbar c(N+1)\frac{n\mathfrak{K}_6}{a^4}\,. \tag{17}$$

The main features of this quantum corrected model:

- anisotropy degrees of freedom produce the ‘radiation-like’ energy density $\rho(a)$; primordial GW?
- curvature-like term kc^2a^{-2}, modifies the usual FRW result;
- new repulsive potential term with a^{-6} generated by the affine CS quantization.

Imposition of Hamiltonian constraint:

The constraint $\check{\mathcal{H}}_{av}(q,p) = 0$ reads

$$\frac{\dot{a}^2}{a^2} + k\frac{c^2}{a^2} + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^6} = \frac{8\pi G}{3c^2}\rho(a)\,, \tag{16}$$

where

$$\mathfrak{s}_P := 2\pi G\hbar\, c^{-3}, \quad k := \frac{\mathfrak{K}_5 n^2}{4}, \quad \rho(a) := \hbar c(N+1)\frac{n\mathfrak{K}_6}{a^4}\,. \tag{17}$$

The main features of this quantum corrected model:

- anisotropy degrees of freedom produce the 'radiation-like' energy density $\rho(a)$; primordial GW?
- curvature-like term $kc^2 a^{-2}$, modifies the usual FRW result;
- new repulsive potential term with a^{-6} generated by the affine CS quantization.

Imposition of Hamiltonian constraint:

The constraint $\check{\mathcal{H}}_{av}(q,p) = 0$ reads

$$\frac{\dot{a}^2}{a^2} + k\frac{c^2}{a^2} + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^6} = \frac{8\pi G}{3c^2}\rho(a)\,, \tag{16}$$

where

$$\mathfrak{s}_P := 2\pi G\hbar\, c^{-3}, \quad k := \frac{\mathfrak{K}_5 n^2}{4}, \quad \rho(a) := \hbar c(N+1)\frac{n\mathfrak{K}_6}{a^4}\,. \tag{17}$$

The main features of this quantum corrected model:

- anisotropy degrees of freedom produce the 'radiation-like' energy density $\rho(a)$; primordial GW?
- curvature-like term kc^2a^{-2}, modifies the usual FRW result;
- new repulsive potential term with a^{-6} generated by the affine CS quantization.

Resolution of singularity

Equation (16) can be rewritten as

$$kc^2 + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^4} - \frac{8\pi G}{3c^2} a^2 \rho(a) \leq 0. \tag{18}$$

which defines allowed values of $a \in [a_-, a_+]$.

The semi-classical trajectories are bounded

from below

$$\frac{\mathfrak{s}_P}{a_-^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 + \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right) \tag{19}$$

and from above

$$\frac{\mathfrak{s}_P}{a_+^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 - \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right). \tag{20}$$

Resolution of singularity

Equation (16) can be rewritten as

$$kc^2 + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^4} - \frac{8\pi G}{3c^2} a^2 \rho(a) \leq 0. \tag{18}$$

which defines allowed values of $a \in [a_-, a_+]$.
The semi-classical trajectories are bounded
from below

$$\frac{\mathfrak{s}_P}{a_-^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 + \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right) \tag{19}$$

and from above

$$\frac{\mathfrak{s}_P}{a_+^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 - \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right). \tag{20}$$

Resolution of singularity

Equation (16) can be rewritten as

$$kc^2 + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^4} - \frac{8\pi G}{3c^2} a^2 \rho(a) \leq 0. \tag{18}$$

which defines allowed values of $a \in [a_-, a_+]$.
The semi-classical trajectories are bounded
from below

$$\frac{\mathfrak{s}_P}{a_-^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 + \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right) \tag{19}$$

and from above

$$\frac{\mathfrak{s}_P}{a_+^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 - \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right). \tag{20}$$

Resolution of singularity

Equation (16) can be rewritten as

$$kc^2 + \mathfrak{s}_P^2 c^2 \frac{\mathfrak{K}_4}{a^4} - \frac{8\pi G}{3c^2} a^2 \rho(a) \leq 0. \tag{18}$$

which defines allowed values of $a \in [a_-, a_+]$.
The semi-classical trajectories are bounded
from below

$$\frac{\mathfrak{s}_P}{a_-^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 + \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right) \tag{19}$$

and from above

$$\frac{\mathfrak{s}_P}{a_+^2} = \frac{2\mathfrak{K}_6}{3\mathfrak{K}_4} n(N+1) \left(1 - \sqrt{1 - \frac{f(\nu)}{(N+1)^2}}\right). \tag{20}$$

Periodicity of trajectories

The semi-classical trajectories are periodic.
The oscillatory period T of the universe is

$$T = \frac{2\mathfrak{t}_P}{\sqrt{\mathfrak{K}_4}}(x_- x_+)^{-3/4}\left(\frac{x_+}{x_-}\right)^{-1/4} E\left(1 - \frac{x_+}{x_-}\right) , \tag{21}$$

where $\mathfrak{t}_P = \sqrt{\mathfrak{s}_P}/c$ is the Planck time, $x_\pm = \mathfrak{s}_P/a_\mp^2$, and E is the complete elliptic integral of the second kind.

Conclusions

Applying

- mixed procedure of quantization (CS+canonical),
- adiabatic approximation to the quantized Hamiltonian $\hat{\mathcal{H}}$,
- constraint $\mathcal{H} = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.

The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into 'primordial GW',
- a curvature-like term kc^2a^{-2} (already present in the classical Bianchi-IX treatment),
- new repulsive potential term $\propto a^{-6}$ generated by our affine CS quantization, responsible for the resolution of the singularity.

Conclusions

Applying

- mixed procedure of quantization (CS+canonical),
- adiabatic approximation to the quantized Hamiltonian $\hat{\mathcal{H}}$,
- constraint $\mathcal{H} = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.

The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into 'primordial GW',
- a curvature-like term kc^2a^{-2} (already present in the classical Bianchi-IX treatment),
- new repulsive potential term $\propto a^{-6}$ generated by our affine CS quantization, responsible for the resolution of the singularity.

Conclusions

Applying

- mixed procedure of quantization (CS+canonical),
- adiabatic approximation to the quantized Hamiltonian $\hat{\mathcal{H}}$,
- constraint $\mathcal{H} = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.
The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into 'primordial GW',
- a curvature-like term kc^2a^{-2} (already present in the classical Bianchi-IX treatment),
- new repulsive potential term $\propto a^{-6}$ generated by our affine CS quantization, responsible for the resolution of the singularity.

Conclusions

Applying

- mixed procedure of quantization (CS+canonical),
- adiabatic approximation to the quantized Hamiltonian $\hat{\mathcal{H}}$,
- constraint $\mathcal{H} = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.
The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into 'primordial GW',
- a curvature-like term kc^2a^{-2} (already present in the classical Bianchi-IX treatment),
- new repulsive potential term $\propto a^{-6}$ generated by our affine CS quantization, responsible for the resolution of the singularity.

Conclusions

Applying

- mixed procedure of quantization (CS+canonical),
- adiabatic approximation to the quantized Hamiltonian $\hat{\mathcal{H}}$,
- constraint $\mathcal{H} = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.
The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into 'primordial GW',
- a curvature-like term kc^2a^{-2} (already present in the classical Bianchi-IX treatment),
- new repulsive potential term $\propto a^{-6}$ generated by our affine CS quantization, responsible for the resolution of the singularity.

Conclusions

Applying

- mixed procedure of quantization (CS+canonical),
- adiabatic approximation to the quantized Hamiltonian $\hat{\mathcal{H}}$,
- constraint $\mathcal{H} = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.

The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into 'primordial GW',
- a curvature-like term $kc^2 a^{-2}$ (already present in the classical Bianchi-IX treatment),
- new repulsive potential term $\propto a^{-6}$ generated by our affine CS quantization, responsible for the resolution of the singularity.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Prospects

Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
- Making predictions for primordial gravitational waves.

Successful quantization of the Bianchi IX model may open the door to the quantization of the BKL scenario.

Thank you!