Phenomenology and relativistic properties of theories with Planck-scale-curved momentum spaces

Spala 1.7.2014

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Planck-scale realm might well be outside the reach of the current "experimental-discovery paradigm" (i.e. particle colliders) but one should seek alternatives.
[proton decay in GrandUnification…Einstein's study of Brownian motion…]

The whole field of quantum-gravity phenomenology is now covered by a "living review": **GAC, Living Reviews in Relativity 16 (2013) 5**

the strategy adopted in the new/emerging field of research in Quantum Gravity Phenomenology is best discussed through an illustrative example

for the illustrative example I use today I like to use the name "phenomenology of minimum-wavelength scenarios" (but you might disagree on the choice of name)

Let us consider the popular idea of the Planck length as the minimum allowed value for wavelengths:

- **suggested by several indirect arguments combining quantum mechanics and GR**
- **found in some detailed analyses of formalisms in use in the study of the QG problem**

But the minimum wavelength is the Planck length for which observer?

GAC, ModPhysLettA (1994)
PhysLettB (1996)

Other studies from the 1990s (mainly from spacetime noncommutativity and LoopQG) provided "theoretical evidence" of Planck-scale modifications of the on-shell relation, in turn inviting us to scrutinize the fate of relativistic symmetries at the Planck scale

These observations first led several researchers to work at the hypothesis that in order to address the quantum-gravity problem one should give up the relativity of observers (preferred-frame picture)

GAC+Ellis+Nanopoulos+Sarkar, Nature(1998)
Alfaro+Tecotl+Urrutia,PhysRevLett(1999)
Gambini+Pullin, PhysRevD(1999)
Schaefer,PhysRevLett(1999)

This would be "Planck-scale <u>broken</u> Lorentz symmetry"

but together with broken Lorentz symmetry one should consider the possibility of "Planck-scale deformations of Poincare' symmetry"
[jargon: "DSR", for "doubly-special", or "deformed-special", relativity]

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

change the laws of transformation between observers so that the new properties are observer-independent

* **a law of minimum wavelength can be turned into a DSR law**
* **could be used also for properties other than minimum wavelength, such as deformed on-shellness, deformed uncertainty relations…**

The notion of DSR-relativistic theories is best discussed in analogy with the transition from Galileian Relativity to Special Relativity

analogy with Galilean-SR transition

introduction to DSR case is easier starting from reconsidering the Galilean-SR transition (the SR-DSR transition would be closely analogous)

Galilean Relativity

on-shell/dispersion relation $E = \frac{p^2}{2m}$ (+m)

linear composition of momenta $p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$

linear composition of velocities $\vec{V} \oplus \vec{V}_0 = \vec{V} + \vec{V}_0$

from Galilean Relativity to Special Relativity

Maxwell theory was not pointing us toward the demise of relativity!
It was pointing to a "relativistic evolution"

The new law concerning the speed of light is not Galilean invariant but is invariant of a theory, special relativity, no less (and no more) relativistic than Galileo's

Relativistic invariance rescued at the "cost" of replacing Galileian boosts with special-relativistic boosts

of course (since c is invariant of the new theory) the special-relativistic boosts act nonlinearly on velocities (whereas Galilean boosts acted linearly on velocities)

and the special-relativistic law of composition of velocities is nonlinear, noncommutative and nonassociative

$$\mathbf{w} = \mathbf{v} \oplus \mathbf{u}$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \frac{1}{1 + \frac{v_1 u_1 + v_2 u_2 + v_3 u_3}{c^2}} \left\{ \left[1 + \frac{1}{c^2} \frac{\gamma_{\mathbf{v}}}{1 + \gamma_{\mathbf{v}}} (v_1 u_1 + v_2 u_2 + v_3 u_3) \right] \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \frac{1}{\gamma_{\mathbf{v}}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\}$$

much undervalued in the (horrible) textbooks we feed our students:

$$\frac{v + u}{1 + (vu/c^2)}$$

Special Relativity (continued)

special-relativistic law of composition of momenta is still linear

$$p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$$

but the on-shell/dispersion relation takes the new form

$$E = \sqrt{p^2 + m^2}$$

and time is relative:

simultaneity of events occuring in the same place (fully coincident events) still is objective

but simultaneity of distant events is "relative", i.e. observer dependent

from Special Relativity to DSR

If there was an observer-independent scale E_P (inverse of length scale ℓ) then, for example, one could have a modified on-shell relation as relativistic law

$$m^2 = \Lambda(E, p; E_P) = E^2 - p^2 - \frac{E}{E_P} p^2 + O\left(\frac{E^4}{E_P{}^2}\right)$$

For suitable choice of $\Lambda(E,p;E_P)$ one can easily have a maximum allowed value of momentum, i.e. minimum wavelength ($p_{max}=E_P$ for $\ell=-1/E_P$ in the formula here shown)

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

it turns out that such laws could still be relativistic, part of a relativistic theory where not only c ("speed of massless particles in the infrared limit") but also E_P would be a nontrivial relativistic invariant

action of boosts on momenta must of course be deformed so that

$$[N_k, \Lambda(E, p; E_P)] = 0$$

then it turns out to be necessary to correspondingly deform the law composition of momenta

$$p_\mu^{(1)} \oplus p_\mu^{(2)} \neq p_\mu^{(1)} + p_\mu^{(2)}$$

(and even the simultaneity of coincident events may no longer be observer-independent)

Let me briefly comment the case of thekappaPOINCARE algebra and the kappaMINKOWSKI noncommutative spacetime

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

Translation generators in kappa-Minkowski:

$$P_\mu \left(e^{ikx} e^{ik_0 t}\right) = k_\mu \left(e^{ikx} e^{ik_0 t}\right)$$

classical action

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0 \quad \Rightarrow \quad e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t} = e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t}$$

!!

then "non-primitive coproduct"

$$P_\mu \left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right) = P_\mu \left(e^{i(k + e^{\lambda k_0} K)x} e^{i(k_0 + K_0)t}\right)$$

$$= \left(k_\mu + e^{-\lambda k_0} K_\mu\right)\left(e^{ikx} e^{ik_0 t} e^{iKx} e^{iK_0 t}\right)$$

$$= \left[P_\mu \left(e^{ikx} e^{ik_0 t}\right)\right]\left(e^{iKx} e^{iK_0 t}\right) + \left[e^{-\lambda P_0}\left(e^{ikx} e^{ik_0 t}\right)\right] P_\mu \left(e^{iKx} e^{iK_0 t}\right)$$

!!

a scheme for relativistic kinematics in kappa-Minkowski (based on nearly two decades of results)

GAC,arXiv:1111.5081,PhysRevD(2012)

on-shell/dispersion relation

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

[notice that this, for $\ell = -1/E_P$, sets maximum momentum E_P]

modified law of composition of momenta

$$(p \oplus_\ell p')_1 = p_1 + e^{\ell p_0} p'_1$$

$$(p \oplus_\ell p')_0 = p_0 + p'_0$$

$$\ell \equiv \lambda \approx \frac{1}{E_P}$$

modified boost action

$$[N, p_0] = p_1$$

$$[N, p_1] = \frac{e^{2\ell p_0} - 1}{2\ell} + \frac{\ell}{2} p_1^2$$

ensures observer-independence of on-shell relation

$$[N, \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2] = 0$$

It was recently realized that this sort of theoretical frameworks a la kappa-Poincare/kappa-Minkowski (with DSR-deformed relativistic laws) may be connected to an old idea advocated by Max Born

one of the first papers on the quantum gravity problem was a paper by Max Born [*Proc.R.Soc.Lond.*A165,29(**1938**)**] centered on the dual role within quantum mechanics between momenta and spacetime coordinates (Born reciprocity)**

$$p_\mu \leftrightarrow x^\mu$$

Born argued that it might be impossible to unify gravity and quantum theory unless we make room for curvature of momentum space

this idea of curvature of momentum space had no influence on quantum-gravity research for several decades, but recently:

momentum space for certain models based on spacetime noncommutativity was shown to be curved

[In particular the kappa-Minkowski results shown earlier are viewed as describing a deSitter momentum space, with the on-shellness relation obtained from the metric as the geodesic distance of a momentum-space point from the momentum-space origin. And the kappa-Minkowski deformed law of composition of momenta introduces a tool for parallel transport on momentum space and therefore an affine connection on momentum space. More on this later]

and perhaps most importantly we learned that the only quantum gravity we actually can solve, which is 3D quantum gravity, definitely has curved momentum space

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

mass of a particle with four-momentum p_μ is determined by the metric geodesic distance on momentum space from p_μ to the origin of momentum space

$$m^2 = d_\ell^2(p,0) = \int dt \sqrt{g^{\mu\nu}(\gamma^{[A;p]}(t))\dot{\gamma}_\mu^{[A;p]}(t)\dot{\gamma}_\nu^{[A;p]}(t)}$$

where $\gamma^{[A;p]}_\mu$ is the metric geodesic connecting the point p_μ to the origin of momentum space

$$\frac{d^2\gamma_\lambda^{[A]}(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu^{[A]}(t)}{dt}\frac{d\gamma_\nu^{[A]}(t)}{dt} = 0$$

with $A^{\mu\nu}{}_\lambda$ the Levi-Civita connection

the affine connection on momentum space determines the law of composition of momenta, through parrallel transport, and it might not be the Levi-Civita connection of the metric on momentum space (it is not in 3D quantum gravity and in all cases based on noncommutative geometry, where momentum space is a group manifold)

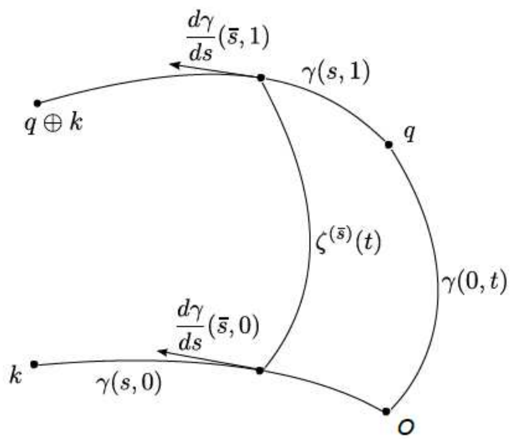


Figure 1. We determine the law of composition of momenta from the affine connection by associating to the points q and k of momentum space the connection geodesics $\gamma^{(q)}$ and $\gamma^{(k)}$ which connect them to the origin of momentum space. We then introduce a third curve $\bar{\gamma}(s)$, which we call the parallel transport of $\gamma^{(k)}(s)$ along $\gamma^{(q)}(t)$, such that for any given value $\bar{s}$ of the parameter s one has that the tangent vector $\frac{d}{ds}\bar{\gamma}(\bar{s})$ is the parallel transport of the tangent vector $\frac{d}{ds}\gamma^{(k)}(\bar{s})$ along the geodesic connecting $\gamma^{(k)}(\bar{s})$ to $\bar{\gamma}(\bar{s})$. Then the composition law is defined as the extremal point of $\bar{\gamma}$, that is $q \oplus_\ell k = \bar{\gamma}(1)$.

“κ-momentum space” [1+1dimensional case]

kappa-Poincare on-shell relation reproduced from geometric interpretation assuming momentum space with de-Sitter metric

$$g^{\mu\nu}(p) = \begin{pmatrix} 1 & 0 \\ 0 & -e^{2\ell p_0} \end{pmatrix}$$

kappa-Poincare composition law reproduced from geometric interpretation assuming (non-Levi-Civita) affine connection such that

$$\Gamma^{\lambda\mu}{}_{\nu} = \ell \delta_0^\lambda \delta_1^\mu \delta_\nu^1$$

kinematics shown earlier, inspired by kappa-Poincare, is fully relativistic (of course DSR-relativistic)

although of a perhaps surprising type (Majid's "back reaction")

GAC+Gubitosi+Palmisano, arXiv:1307.7988

In a relativistic theory there must be some generators that leave the on-shell relation invariant. For the kappa-kinematics shown earlier these are

$$\begin{cases} \tilde{N}_0(p) = p_1 \\ \tilde{N}_1(p) = \left(\frac{1}{2\ell}\left(1 - e^{-2\ell p_0}\right) - \frac{\ell}{2}p_1^2\right) \end{cases}$$

From the generators one then gets finite symmetry transformations

$$p_\mu \to \Lambda^\xi_\mu(p)$$

This said about on-shellness next task is conservation laws, which must be relativistically convariant

For the kappa-kinematics discussed above the composition law is

$$\begin{cases} (q \oplus_\ell k)_0 = q_0 + k_0 \\ (q \oplus_\ell k)_1 = q_1 + k_1 e^{-\ell q_0} \end{cases}$$

And only way to make it consistent with action of boosts is to have that the action of boosts on composition of momenta is characterized by "Majid's back-reaction":

$$B^\xi \triangleright (q \oplus_\ell k) = \tilde{\Lambda}^\xi(q) \oplus_\ell \tilde{\Lambda}^{\xi e^{-\ell q_0}}(k)$$

kappa-kinematics is fully relativistic and noncommutativity of composition law is not a severe challenge for phenomenology

Still it would be interesting if there were theories with a minimum wavelength and with commutative composition law

A simple path for obtaining such a theory is provided by using again the de Sitter momentum space

GAC+Gubitosi+Palmisano, arXiv:1307.7988

$$g^{\mu\nu}(p) = \begin{pmatrix} 1 & 0 \\ 0 & -e^{2\ell p_0} \end{pmatrix}$$

$$\begin{cases} \tilde{N}_0(p) = p_1 \\ \tilde{N}_1(p) = \left(\frac{1}{2\ell}\left(1 - e^{-2\ell p_0}\right) - \frac{\ell}{2}p_1^2\right) \end{cases}$$

A commutative composition law is obtained using the geometric-interpretation prescription now replacing the kappa-connection with the Levi-Civita connection of the deSitter metric. To quadratic order one then gets

$$\begin{cases} (q \oplus_\ell k)_0 = q_0 + k_0 - \ell q_1 k_1 + \frac{\ell^2}{2}\left[-q_1 k_1 (q_0 + k_0) + q_0 k_1^2 + q_1^2 k_0\right] \\ (q \oplus_\ell k)_1 = q_1 + k_1 - \ell (q_0 k_1 + q_1 k_0) + \frac{\ell^2}{2}\left[(q_0 k_1 + q_1 k_0)(q_0 + k_0) + q_1 k_1^2 + q_1^2 k_1\right] \end{cases}$$

the compatibility of this composition law with the action of boosts is achieved without any "back reaction"

$$B^{\xi} \triangleright (q \oplus_{\ell} k) = \tilde{\Lambda}^{\xi}(q) \oplus_{\ell} \tilde{\Lambda}^{\xi}(k)$$

Appreciating that some results on quantum-spacetime/quantum-gravity theories can be reformulated as properties of curved momentum spaces is proving to be very valuable also for phenomenology

Much studied opportunity for phenomenology comes from fact that some pictures of quantum spacetime predict that the speed of photons is energy dependent.

Interpretation of this energy dependence has been a key challenge and the source off much controversy, but we now understand it in very intuitive terms!!!
as dual redshift on Planck-scale-curved momentum spaces:

these results so far are fully understood for the case of
[maximally symmetric curved momentum space] ⊗ [flat spacetime]

it turns out that there is a duality between this and the familiar case of
[maximally-symmetric curved spacetime] ⊗ [flat momentum space]

In particular, ordinary redshift in deSitter spacetime implies in particular that massless particles emitted with same energy but at different times from a distant source reach the detector with different energy
dual redshift in deSitter momentum space implies that massless particles emitted simultaneously but with different energies from a distant source reach the detector at different times

GAC+Barcaroli+Gubitosi+Loret, Classical&QuantumGravity30,235002 (2013)
GAC+Matassa+Mercati+Rosati, PhysicalReviewLetters106,071301 (2011)

dual redshift on Planck-scale-curved momentum spaces **(but with flat spacetime) produces time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)**

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent "physical velocity" of ultrarelativistic particles

$$\mathrm{v} = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

<u>these are very small effects but (at least for the case n=1) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!</u>

GRBs are ideally suited for testing this:
cosmological distances (established in 1997)
photons (and neutrinos) emitted nearly simultaneously
with rather high energies (GeV…..TeV…100 TeV…)

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC, NaturePhysics10,254(2014)

problem:

solid theory is for (curved momentum space and) flat spacetime

phenomenological opportunities are for propagation over cosmological distances, whose analysis requires curved spacetime

study of theories with both curved momentum space and curved spacetime still in its infancy

GAC+Rosati, PhysRevD86,124035(2012)
KowalskiGlikman+Rosati,ModPhysLettA28,135101(2013)
Heckman+Verlinde,arXiv:1401.1810(2014)

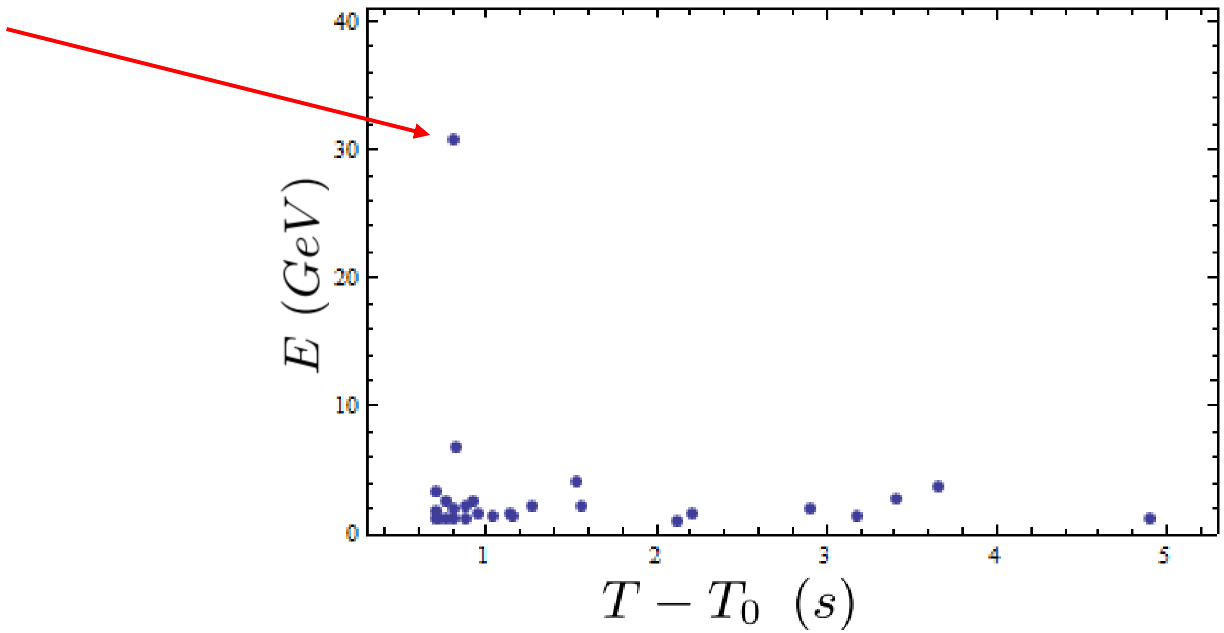
Jacob and Piran [JCAP0801,031(2008)] used a compelling heuristic argument for producing a formula of energy-dependent time delay applicable to FRW spacetimes, which has been the only candidate so far tested

$$\Delta T = -s_{\pm} \frac{E}{M_{QG}} \frac{c}{H_0} \int_0^z d\zeta \frac{(1+\zeta)}{\sqrt{\Omega_\Lambda + (1+\zeta)^3 \Omega_m}}$$

where as usual H_0 is the Hubble parameter, Ω_Λ is the cosmological constant and Ω_m is the matter fraction.
But Jacob-Piran formula is surely not the most general possibility….

clearest example of tests of energy dependence of the speed of photons:

GRB090510



a test with accuracy of about one part in 10^{20}!!!

“bring-home points”:

- **3D quantum gravity is a DSR-relativistic theory with curved momentum space**
- **Some noncommutative spacetimes lead to theories which are DSR-relativistic and have curved momentum space**
- **Energy dependence of speed of photons is nothing else but dual redshift on a curved momentum space**
- **Dual redshift can be tested experimentally with Planck-scale sensitivity**

ctcqg2014.relativerest.org [Sapienza Univ of Rome, September 8-12]
big quantum-gravity conference with talks by leaders of most QG research lines!!!!