An approach to initial conditions in general relativity

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Tafel J. 2014, An approach to initial conditions in general relativity, sent to *Class. Quantum Grav.*, arXiv: 1405.1202

Tafel J. and Jóźwikowski M. 2014, New solutions of initial conditions in general relativity, *Class. Quantum Grav.* **31** 115001, arXiv:1312.7819

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Notation: $T^{ij} = K^{ij} - Hg^{ij}$

The conformal approach (Lichnerowicz, Choquet-Bruhat, York)

$$g'_{ij} = \psi^4 g_{ij}, \quad K'_{ij} = \psi^{-2} K_{ij} + \frac{1}{3} H' g_{ij} \ , \quad {K^i}_i = 0$$

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An exception: if $H' = const$ then

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(the momentum constraint with $H = 0$). Thus, it makes sense to solve the momentum constraint first, provided that H can be set to zero.

The momentum constraint in the Gauss coordinates

Initial metric

$$g = g_{ab}dx^a dx^b + d\varphi^2 \ , \quad \varphi = x^3 \ .$$

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$$(|\tilde{g}|T^{33})_{,3} = |\tilde{g}|(g_{ab,3}T^{ab} - T^{3a}{}_{|a}) \ ,$$

where

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Problems with integration if $H = 0$ or ∂_φ is a symmetry.

Arbitrary foliation of initial manifold

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$$e_a = \partial_a - \beta_a \partial_3 \ , \quad e_3 = \partial_3 \ .$$

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$$(|\tilde{g}|T_3{}^3)_{,3} - \frac{1}{2}|\tilde{g}|g_{ab,3}T^{ab} = 2|\tilde{g}|\beta_{a,3}T_3{}^a - \frac{1}{\alpha}e_a(\alpha|\tilde{g}|T_3{}^a)$$

$$(\alpha T_a{}^b)_{|b} - |\tilde{g}|^{-1}(\alpha|\tilde{g}|T_a{}^b\beta_b)_{,3} = (e_a\alpha - \alpha\beta_{a,3})T_3{}^3 - \frac{1}{2}\alpha(g_{bc,3}T^{bc})\beta_a + \alpha\lambda\eta_{ab}T_3{}^b - |\tilde{g}|^{-1}(\alpha|\tilde{g}|T_a{}^3)_{,3} \ ,$$

where η^{ab} is the Levi-Civita tensor and $\lambda = \eta^{ab}e_a\beta_b$.

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Functions equivalent to $K_{ij} - \frac{1}{3}Hg_{ij}$:

$$U = \frac{1}{2}\alpha(T_{11} - T_{22}) + i\alpha T_{12} \ , \quad V = \alpha(T_{13} + iT_{23}) \ , \quad W = {T_3}^3 + \frac{2}{3}H \ .$$

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A function of V and the metric (no U and W)

$$E = \rho\alpha^{-1} Re(2\beta_{,3} V - \partial V) + \frac{2}{3}\rho^3 H_{,3} \ , \quad \partial = \partial_\xi - \beta\partial_3 \ .$$

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A function of V, W and the metric (no U)

$$F = \frac{1}{2}\rho^2\alpha^{-2}\bar{\partial}(\alpha^3 W) - \frac{3}{2}\rho^2\alpha W\bar{\beta}_{,3} - iIm(\partial\bar{\beta})V - (\alpha^{-2}\rho^2 V)_{,3} + \frac{2}{3}\rho^2\alpha\bar{\partial}H \ .$$

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The momentum constraint is equivalent to

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For analytic data

$$U = \bar{\chi}_{,3}\Big(\int_{\xi_0}^{\xi} d\xi' \ \frac{F}{\bar{\chi}_{,3}} + h(\bar{\xi}, \bar{\chi})\Big) \ ,$$

where $\beta = \bar{\chi}_{,\xi}/\bar{\chi}_{,3}$ *and the integrand is considered as a function of* ξ', $\bar{\xi}$ *and* $\bar{\chi}$.

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In the nonanalytic case with $\beta = 0$

$$U = -\frac{1}{4\pi}\int d^2x' \frac{F(x'^a, \varphi)}{(\bar{\xi}' - \bar{\xi})} + h(\bar{\xi}, \varphi) \ .$$

Comments

If $U = \hat{U}_{,3}$ and $F = \hat{F}_{,3}$ then

$$\partial \hat{U} = \hat{F} \ .$$

Let the CR structure corresponding to ∂ be realizable: $\partial \bar{\chi} = 0$ and $\chi \neq f(\xi)$. Equivalently

$$\beta = \frac{\bar{\chi}_{,\xi}}{\bar{\chi}_{,3}} \ .$$

Then

$$\hat{U} = \int_{\xi_0}^{\xi} d\xi' \ \hat{F}(\xi', \bar{\xi}, \bar{\chi}) + \hat{h}(\bar{\xi}, \bar{\chi})$$

and taking the φ-derivative yields U.

Formula for $\beta = 0$ follows from the fundamental solution of the Laplace equation.

Then

$$\partial V = 0 \ .$$

Equivalently, in terms of real fields and coordinates,

$$T_3{}^a = \alpha^{-1}\eta^{ab}\omega_{,b} \ .$$

Other components of T^{ij} are given in quadrature. In generic case they can be found explicitly but then $H = 0$ is a differential equation.

Families of data generalizing the Kerr data can be constructed (up to a knowledge of ψ).

There exists a class of data (defined in quadrature) which satisfy all constraints but they are not asymptotically flat.

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Theorem (D. Maxwell)

Let (S, g) be a complete Riemannian manifold without boundary and let (g, K) be asymptotically flat data of class $W_\delta^{k,p}$.

There exists a unique solution of the Lichnerowicz equation such that $\psi > 0$ and $(\psi - 1) \in W_\delta^{k,p}$ if and only if

$$\|f\|_{L^6}^2 \leq \lambda\Big(\int_S (8|\nabla f|^2 + Rf^2)dv_g\Big)\ , \quad \lambda > 0$$

for all $f \in C_c^\infty$.

Full system of constraints for $H' \neq const$

$$\triangle\psi = \frac{1}{8}R\psi - \frac{1}{8}K_{ij}K^{ij}\psi^{-7} + \frac{1}{8}H'^2\psi^5$$

$$(\rho^3 W)_{,3} = \frac{2}{3}\psi^6\rho^3 H'_{,3} + E_0$$

$$U_{,\xi} - (\beta U)_{,3} = \frac{2}{3}\psi^6\rho^2\alpha\bar{\partial}H' + F_0$$

where E_0 and F_0 denote E and F with $H = 0$.

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Calculate H from the Hamiltonian constraint

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Substitute H and ${T_3}^3$ into the equation for U. If $\beta = 0$ it reads

$$U_{,\xi} - \rho^2 (fU\bar{U})_{,\bar{\xi}} = h \ .$$

Functions f and h are known and coordinate φ appears as a parameter.

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Problems with the existence theorems unless $K(n,n) = 0$ and with an extension of initial data through the boundary (some solutions are constructed in the paper with Jóźwikowski).

An approach without boundary

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Initial metric induced by the Kerr solution on $t = const$

$$g = \rho^2 \Delta^{-1} dr^2 + \rho^2 d\theta^2 + \rho^{-2} \Sigma^2 \sin^2 \theta d\varphi^2$$

where

$$\rho^2 = r^2 + a^2 \cos^2 \theta \ , \quad \Delta = r^2 - 2Mr + a^2 \ , \ \Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta \ .$$

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Nonvanishing components of T^{ij}:

$$T_3{}^a = \alpha^{-1}\eta^{ab}\omega_{,b} \ ,$$

$$\omega = 4aM\rho^{-2}[2(r^2 + a^2) + (r^2 - a^2)\sin^2\theta]\cos\theta \ .$$

Conformally flat complex coordinate: $\xi = (\tilde{r} + i\theta)/2$, where

$$r = M + \sqrt{M^2 - a^2}\cosh\tilde{r} \ , \ \tilde{r} \in [-\infty, \infty]$$

($\tilde{r} = 0$ is the external Kerr horizon).

Properties of the data

- $H = 0$
- Invariance under $\tilde{r} \to -\tilde{r}$
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Solving the momentum constraint with respect to U and W under these assumptions is practically impossible because of the condition $T^{ij}n_i n_j = 0$. Let us try to solve them with respect to V.

Proposition

Let

$$g = \rho^2 d\xi d\bar{\xi} + \alpha^2 d\varphi^2 \ , \quad \rho_{,3} = \alpha_{,3} = H = 0$$

and functions ρ, α *and* T^{ij} *be even functions of* $\tilde{r} = \frac{1}{2} Re\xi$.

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If ReU *and* ${T_3}^3$ *satisfy*

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then momentum constraint can be solved with respect to V. If there exists a unique solution ψ of the Lichnerowicz equation then the initial data admit a MOTS at $\tilde{r} = 0$.

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This equation is hyperbolic and coordinate φ enters as a parameter.

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- The Gauss coordinates are not convenient from the point of view of the Hamiltonian constraint.
- For analytic data or reduced number of free functions the momentum constraint can be solved in quadrature and the conformal method can be applied if $H = 0$.
- The algebraic method of solving the Hamiltonian constraint and the construction of a generalized Kerr data with MOTS are (perhaps) worth studying.
- More work needed to include multiple horizons.